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Deschauer, Stefan (D-TUD)

★Die große Arithmetik aus dem Codex Vind. phil. gr. 65. (German) [The great arithmetic of the Codex Vind. phil. gr. 65]

Eine anonyme Algorismusschrift aus der Endzeit des byzantinischen Reiches.

[An anonymous algorism manuscript from the end of the Byzantine Empire].

Textbeschreibung, Transkription, Teilübersetzung mit Kommentar, Fachsprache, Vokabular, Metrologie.

[Textual description, transcription, partial translation with commentary, terminology, vocabulary, metrology].

Denkschriften Österreichische Akademie der Wissenschaften, Philosophisch-Historische Klasse [Memoirs of the Austrian Academy of Sciences, Philosophy-History Section], 461.

Veröffentlichungen zur Geschichte der Naturwissenschaften, Mathematik und Medizin [Publications on the History of Natural Sciences, Mathematics and Medicine], 67.

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The volume under review contains an edition and a partial translation of the “first book” of a Byzantine “algorism” from 1436 (a “second book” deals with root extraction, algebra in Italian abacus style, and geometry, a future description and study of which are planned by Deschauer). “Algorisms” designate the European medieval and Renaissance introductions to the use of Hindu-Arabic numerals. They fall into two main groups—the normal Latin type, which only teaches the notation and the calculational algorithms (henceforth “restricted”); and the vernacular type (primarily Italian), which after the introduction of the new numerals and their use go on to present operations with “vulgar” (not sexagesimal) fractions, and then deal with commercial arithmetic. The vernacular type may skip the explanation of the calculational algorithms (e.g., Jacopo da Firenze), or only present those for multiplication and division (e.g., Giovanni de’ Danti).

The treatise here edited contains a full presentation of the Hindu-Arabic numerals, but it also teaches the arithmetic of vulgar fractions as well as select commercial topics; it thus transgresses the dichotomy. It is one of rather few Byzantine treatises belonging to one or the other kind. Firstly, there are two restricted algorisms: one, anonymous, from 1252 [A. Allard, *Rev. Hist. Textes* 7 (1978), no. 1977, 57–107], the other, written by Maximos Planudes some decades later, drawn from this and other sources [A. Allard, *Maxime Planude—Le grand calcul selon les Indiens*, Univ. Catholique Louvain, Louvain-La-Neuve, 1981]; in part derived from these are Rhabdas’ “Arithmetical letters” from 1341 [P. Tannery, in *Mémoires scientifiques IV*, 61–198, Édouard Privat, Toulouse, 1920].

Then there are, beyond the present one and its companion piece (about which see below), two presentations of commercial arithmetic: one is from the early fourteenth century [K. Vogel, *Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts: Text, Übersetzung und Kommentar*, Wiener Byzantinistische Studien, Band VI. Österreichische Akademie der Wissenschaften. Kommission für Byzantinistik. Institut für Byzantinistik der Universität Wien, In Kommission bei Hermann Böhlaus Nachf., Vienna, 1968; MR0250826 (40 #4058)] and contains only problems, no algorism, and the other, written by Rabbi Elia Misrachi c. 1500 [G. Wertheim, *Die Arithmetik des Elia Misrachi*, F. Vieweg u. Sohn, Braunschweig, 1896; JFM 27.0031.02], has a full algorism with fractions and commercial arithmetic.

The latter two are independent of each other, and also of the restricted algorithms. The treatise edited by Deschauer appears to be independent of all of them. Thus, it not only fills a lacuna in a “tradition”, it enhances our awareness of the richness of the complete record (and of how much has probably been lost, warning against any peremptory judgments).

Deschauer has edited a treatise contained in Cod. Vind. phil. gr. 65. This codex also contains a problem collection [H. Hunger and K. Vogel, *Ein byzantinisches Rechenbuch des 15. Jahrhunderts*, 100 Aufgaben aus dem Codex Vindobonensis Phil. Gr. 65. Text, Übersetzung und Kommentar, mit 24 Tafeln. Österreichische Akademie der Wissenschaften, Philosophisch-Historische Klasse, Denkschriften, 78. Bd., 2. Abhandlung, Hermann Böhlaus Nachf., Kommissionsverlag der Österreichischen Akademie der Wissenschaften in Wien, Graz, 1963; [MR0155738 \(27 #5672\)](#)], which, as Deschauer can now convincingly argue (p. 12), was later than the treatise here edited, and written elsewhere by a different scribe.

The part of the codex edited by Deschauer was already edited in 2006, but without any translation and apparently for linguistically interested Byzantinists only. For any historian of mathematics, familiar or unfamiliar with late medieval scholarly and vernacular Greek, Deschauer’s edition with partial translation and ample commentary is thus unavoidable.

The algorism part of the treatise is likely to be independent of the anonymous one from 1252 and that of Planudes. It does not use the Hindu-Arabic shapes but represents the digits 1 through 9 by the familiar “Milesian” letter symbolism, $\alpha = 1$, $\beta = 2$, etc. (Similarly, Misrachi sometimes used Hebrew letters—but not systematically.) Zero is represented by an idiosyncratic invention, looking like an “h” rotated 180°. Moreover, the treatise introduces and uses decimal fractions. Deschauer supposes, with Hunger and Vogel, that this was a borrowing from al-Kāshī’s recent work, mediated by the Ottoman Turks, who already occupied much of the former Byzantine area. (In the reviewer’s opinion, an independent calque of the well-known sexagesimal fractions should not be excluded—decimal fractions had already been invented by Immanuel Bonfils of Tarascon around 1350 and were to be reinvented by Stevin, while place-value fractions with an arbitrary factor of descent had been examined by Jordanus of Nemore earlier in the thirteenth century.)

In general, the treatise is generously provided with (generally adequate) pedagogical explanations. It does contain some blunders, but in general the mathematical level is good. The problems presented are either linked to commercial practice or of recreational type, but as pointed out by Deschauer (p. 353) the treatise as a whole is systematic in character and no handbook for merchants or their clerks.

Beyond the links pointed out by Deschauer, one should be taken note of because of its general significance. The rule of three is introduced by means of questions like these: “if three become 4, how much will 5 become?” and “if eight times 8 would not become 64 but grew illicitly and became 100, how much would in the same ratio twelve times 12 become?” Deschauer sees this (p. 268) as an unusually theoretical approach built on the classical theory of proportions. A reference to the Euclidean theory is indeed found in many Arabic works, but not here. Instead, such counterfactual statements serve in all Romance Ibero-Provencal commercial arithmetics until 1500 to introduce the rule of three, while Fibonacci and Italian abacus books present them either as a distinct topic or after a sequence of rule-of-three examples of goods versus money or one type of coin versus another type; no Arabic work seems to have them. It remains possible, as suggested by Deschauer (p. 351), that the Byzantine author built, in the main, on an unknown Italian source; but he must have then combined even more freely with other

material than suggested by Deschauer; other readers are likely to discover further links that have escaped both Deschauer and the present reviewer.

Jens Høyrup

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